RM-6573/200

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B. E. II (Sem. IV) (Mech.) Examination

May / June – 2010

Maths - III

Time : Hours] [Total Marks :

Instruction :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. 2 (Sem. 4) (Mech.)**

Name of the Subject :
☛ **Maths - 3**

☛ Subject Code No. : **6 5 7 3** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

Q-1 a) Attempt the following **[10]**

i) Evaluate $\int_{\pi/2}^{1}\int_{1}^{2} x \cos xy \, dydx$

ii) Define periodic function and state the period of $\sin 2x$.

iii) Show that the vector $\vec{F} = (z + \sin y)i + (x\cos y - z)j + (x - y)k$ is irrotational vector.

iv) For any closed surface σ show that $\iint_{\sigma} curl \overline{F}.\hat{n} \, ds = 0$

v) Show that $grad(r) = \frac{\vec{r}}{r}$. Where, $\vec{r} = xi + yj + zk$.

b) Attempt any two of the following. **[10]**

i) Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1+x^2}} \frac{dydx}{\sqrt{1+x^2+y^2}}$

ii) Evaluate $\iint_{R}(2x - y^2)\, dA$ over the triangular region R enclosed by $y = -x+1, y = x+1 \text{ and } y = 3$.

iii) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and by the planes $y + z = 4 \text{ and } z = 0$.

Q-2 a) Verify Green's theorem for $\oint_{C}\left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$ around the boundary C of the region $x = 0, y = 0 \text{ and } x + y = 1$. **[6]**

b) Attempt any three of the following. **[9]**

i) A vector field is given by $\vec{F} = (\sin y)i + x(1 + \cos y)j$. Evaluate the line integral $\int_{C} \vec{F}.\vec{dr}$ over the circular path $x^2 + y^2 = a^2$ and $z = 0$.

ii) Show that $\vec{F} = (6xy + z^3)\, i + (3x^2 - z) j + (3xz^2 - y) k$ is irrotational & find its scalar potential.

iii) Find a vector normal to the surface $x^2y + zx$ at the point $(1,0,2)$ in the direction of $i - j + 2k$.

iv) Find the directional derivative of $f(x,y,z) = 2xyz + z^2$ at the point $(1,-1,3)$ in the direction of the outer normal $i + 2j + 2k$.

Q-3 a) Define periodic function. Obtain Euler's formulae for the periodic function in the interval $0 < x < 2\pi$. **[5]**

b) Attempt any two of the following. **[10]**

i) Obtain Fourier series expansion for $f(x) = x^2, -\pi \le x \le \pi$ .Hence show that

a) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} = \frac{\pi^2}{12}$

ii) Obtain Fourier series expansion for the function $f(x) = x \sin x$ in the interval $0 < x < 2\pi$.

iii) Obtain half-range cosine series for $f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l-x), & l/2 \le x \le l \end{cases}$

Section-II

Q-4 a) Attempt the following : **[10]**

i) Evaluate $\int_0^{\pi/2} \cos^{5/2} x \, dx$

ii) Prove that $erf(x) + erf_c(x) = 1$.

iii) Obtain one L.I. solution of the PDE $zp - zq = z^2 + (x+y)^2$

iv) Write any two properties of beta function.

v) Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$

b) Attempt the following. **[10]**

i) State and prove relation between beta and gamma function. **[04]**

ii) Prove that $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx = \frac{n!}{(m+1)^{n+1}}$. **[03]**

iii) Prove that $\beta(m,n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$ **[03]**

Q-5 a) Solve the following One-dimensional Heat equation **[08]**

$u_t = c^2 u_{xx}; t > 0 \; and \; 0 < x < l$

$u(0,t) = 0 = u(l,t); t > 0$

$u(x,0) = f(x)$

Using separation of variables method.

b) Attempt any one of the following. **[07]**

i) A taut string of length *l* has its ends *x=0* and *x=l* fixed. The midpoint is stretched to a small height and released from rest at time t=0. Find the deflection $u(x,t)$.

ii) Solve the following boundary value problem

$u_{xx} + u_{yy} = 0$ with the conditions

$u(0,y) = u(a,y) = 0 \; for \; 0 \le y \le b$ and

$u(x,0) = 0 \; and \; u(x,b) = f(x), 0 \le x \le a$.

Q-6 a) **Attempt any two of the following.** **[06]**

i) Solve $\left(y^2+z^2\right)p-xyq+xz=0$

ii) Solve $x^2\left(y-z\right)p+y^2\left(z-x\right)q=z^2\left(x-y\right)$

iii) Solve $xp+yq=x$.

b) **Attempt any three of the following.** **[09]**

i) A car-hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and proportion of days on which some demand is refused.
$(e^{-1.5}=0.2231)$

ii) A sample of 100 battery cells tested to find the length of life produced the following results:

$\bar{x}=12\ hours, \sigma=3\ hours.$

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

i. Less than 6 hours

ii. More than 15 hours

[S.N.V. (2)=0.4772 & S.N.V. (1)=0.3413]

iii) A random sample of 900 members has a mean 3.4 cms. Can it be reasonably regarded as a sample from a large population of mean 3.2 cms. and standard deviation 2.3 cms.?
[z at 5% level of significance is 1.96]

iv) A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis it is a random sample from a normal population with mean 45 units. [t at 5% level of significance and 19 d.f. is $t_{0.05}=2.09$